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The potential of director theory for modelling blood flow in the cardiovascular system

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Introduction

We are looking at using director theory (also known as Cosserat theory) as an alternative to classical 1D models for arterial modelling of the human cardiovascular system. The motivation behind applying director theory to the modelling of the human cardiovascular system is to retain good accuracy while maintaining a low computational cost, in the hope that this would allow modelling techniques to be a viable option in a clinical setting. While it is possible to reconstruct 3D computational models of individual patients using non-invasive medical imaging techniques, only a section of this can be used for 3D CFD simulations, otherwise the computational effort becomes too great. This can be coupled with 1D modelling for the arterial branching.

Method

The director theory is hierarchical, so the accuracy can be improved, at the cost of the simplicity of the equations. The system of equations is closed at each order, meaning that no assumptions need to be made about the form of the nonlinear and viscous terms. The theory allows for the description of curvature, torsion as well as non-circular cross-sections. This results in a more accurate solution of the flow field as compared to classical 1D models which are effectively straight rods. Preliminary discussion and comparison of director theory to classical 1D models are outlined in Robertson and Sequeira [1].

Results

The first results we obtained by applying director theory to fluid, following the approach of Caulk and Naghdi [2], were for Poiseuille flow and steady swirling flow in a straight pipe of constant radius. The steady swirling flow is shown in Fig. 1. Being able to recover a swirling flow with the director approach demonstrates its advantage over the classical 1D approach, which can only capture coaxial ow. We continue to develop the theory and intend to show ow solutions for curved pipes of varying cross section.

$$
\begin{aligned}
\mathbf{v} = {} & (\phi^2 - (x_1^2 + x_2^2))x_2 A \exp\left(\frac{3\rho\phi v_0 - (9\rho^2\phi^2 v_0^2 + 1600\mu^2)^{1/2}}{10\mu\phi} x_3\right)\mathbf{e}_1 \\
& - (\phi^2 - (x_1^2 + x_2^2))x_1 A \exp\left(\frac{3\rho\phi v_0 - (9\rho^2\phi^2 v_0^2 + 1600\mu^2)^{1/2}}{10\mu\phi} x_3\right)\mathbf{e}_2 \\
& + \left(1 - \frac{x_1^2 + x_2^2}{\phi^2}\right) v_0 \mathbf{e}_3
\end{aligned}
$$

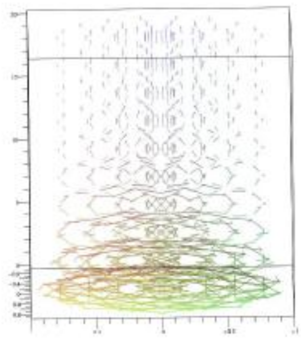

Figure 1: Equation and 3D side on plot of the steady swirling flow, with coefficients $\mu = 0.01$, $\rho^* = 1$, $\phi = 1$, $v_0 = 1$, $A = 5$.

Discussion

We show in this abstract the potential for director theory for modelling the cardiovascular system, without resorting to the over-simplifications required by classical 1D models. This includes retaining the geometrical features of vessels, such as cross-sectional shape, curvature and torsion, leading to more accurate description of the 3D flow field despite using a 1D model.

Acknowledgements

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References

[1] Robertson AM, Sequeria A. A director theory approach for modeling blood ow in the arterial system: An alternative to classical 1D models. Mathematical Models and Methods in Applied Sciences. Vol 15, No.6 871-906, 2005

[2] Caulk DA, Naghdi PM. Axisymmetric motion of a viscous fluid inside a slender surface of revolution. Journal of Applied Mechanics. Vol 54, 190-196, 1993.

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